

#### BACKGROUND

A check between measured and theoretical towcable characteristics is valuable. This check cannot easily be made if a large number of calculations must be performed.

The integral expressions for towcable geometry and forces that are derived in reference (a) include four non-dimensional cable functions,  $\tau$ , (For definitions and symbols see Glossary of Terms.)  $\xi$ ,  $\eta$ , and  $\sigma$ . The cable functions for any one towcable depend upon the angle  $\phi$  at any point along the towcable and two ratios of cable parameters. One ratio, f, expresses the relative resistance of the cable (i.e., ratio of the tangential to the normal cable drag coefficients), and the other ratio, w, expresses the critical angle of tow for the towcable.

Reference (c) was written to simplify the use of the cable functions contained in the tables of reference (a); the tabular functions of reference (a) are graphed with an accuracy of four per cent; numerical examples are also given. 254299

The present memorandum should further aid the user of references (a) or (c), since it provides a nomograph that will permit swifter computations to be made.

#### DESIGN OF THE NOMOGRAPH

To design the nomograph, the following information is needed:

- the equations relating the variables;
- b. the range of the variables:
- c. the determination of the dependent variables;
- d. the identification of the type of equation; and,
- the scale modulus or unit of representation to be used in making the desired variable scales fit on the sheet of paper.

Information for the design of the nomograph was obtained from reference (d).

## a. Equations Relating the Variables

Four basic equations are involved in the nomograph design. The equations relate the non-dimensional cable functions  $\tau, \sigma, \xi$ , and  $\eta$ the towcable tension, length, horizontal projection of length, and vertical projection of length, respectively. These equations are from reference (a):

#### Towcable Tension

$$T = T_o \frac{\tau}{\tau_o}$$

$$S = \frac{T_o (\sigma - \sigma_o)}{R \tau_o}$$

Towcable Length

$$S = \frac{T_o (\sigma - \sigma_o)}{R \tau_o}$$
 (2) O cp V R

Towcable Horizontal Projection

$$X = \frac{T_o(\xi - \xi_o)}{R\tau_o}$$
N=D Calle (m)
(3) Dep V

Towcable Vertical Projection

$$Y = \frac{T_o (\eta - \eta_o)}{R \tau_o}$$
 (4) Dep V.

See Glossary of Terms for the definitions and symbols used.

## b. Range of the Variables

From references (a) and (c) and discussions with members of the Laboratory, the following ranges were established as those that are representative in the design of a Variable Depth Sonar system.

Towcable tensions (pounds):  $10 \le T \le 1,000,000$ 

Towcable lengths (feet):  $1 \le S \le 1,000$ 

Cable Friction (pounds/foot):  $0.1 \le R \le 1,000$ 

Tension Cable Function:  $1 \le \tau \le 10$ 

Length Cable Functions:  $0 \le (\sigma - \sigma_0) \le 10$ 

Horizontal Projection Cable Functions:  $0 \le (\xi - \xi_0) \le 10$ 

Vertical Projection Cable Functions:  $0 \le (\eta - \eta_0) \le 10$ 

Trail Distances (feet):  $1 \le X \le 1,000$ 

Depth Distances (feet):  $1 \le Y \le 1,000$ 

# c. Determination of the Dependent Variables

From equations (1) through (4), it is seen that two terms,  $\tau$  and  $T_{o}$ , appear in each equation and that the cable resistance, R appears in three equations. The other variables in each equation are the cable function itself,  $\tau$ , in equation (1), or the difference between the function at an arbitrary point of interest and at the towpoint, e.g.,  $(\sigma-\sigma_{o})$ ;  $(\xi-\xi_{o})$ ; and  $(\eta-\eta_{o})$ .

In order to determine which of the above variables should be the dependent variable, the test product is calculated, (see reference (d)). The test product is defined as the "... product of the coefficient of a variable in its plotting equation and of the extent of its assigned range..." Utilizing equations (1) through (4), the ranges established above and the test product, it has been determined that for equation (1), the dependent variable is T, and for equations (2), (3), and (4), the dependent variable is R.

## d. Identification of the Type of Equation

With the two dependent variables, T and R, equations (1) through (4) are next written in terms of the dependent variables. In addition, the equations are written in logarithmic form, in order to construct the nomograph. Rewriting equations (1) through (4):

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$$\log T = \log T_0 + \log \tau - \log \tau_0 \tag{1A}$$

$$\log R = \log T_o + \log (\sigma - \sigma_o) - \log \tau_o - \log S$$
 (2A)

$$\log R = \log T_0 + \log (\xi - \xi_0) - \log \tau_0 - \log X$$
 (3A)

$$\log R = \log T_0 + \log (\eta - \eta_0) - \log \tau_0 - \log Y$$
 (4A)

Equations (1A) through (4A) are of a form referred to as "Equations with Four or More Variables," (see reference (d)). These types of equations have significance for nomographic plotting in indicating how auxiliary non-graduated scales should be constructed.

The auxiliary scales are obtained as follows:

Let 
$$\log T_0 - \log \tau_0 = Q$$
.

Inserting the above expression into equations (IA) through (4A), one obtains:

$$\log T = \log \tau + Q \tag{1B}$$

$$\log R = \log (\sigma - \sigma_0) - \log S + Q \tag{2B}$$

$$\log R = \log (\xi - \xi_0) - \log X + Q \tag{3B}$$

$$\log R = \log (\eta - \eta_0) - \log Y + Q \qquad (4B)$$

Letting:

$$P_1 = Q - \log S$$

$$P_2 = Q - \log X$$

Equations (1B) through (4B) are written as:

$$\log T = \log \tau + Q \tag{5}$$

$$\log R = \log (\sigma - \sigma_0) + P_1 \tag{6}$$

$$\log R = \log (\xi - \xi_0) + P_0 \tag{7}$$

$$\log R = \log (\eta - \eta_o) + P_3 \tag{8}$$

Equations (5) through (8) have been calibrated in Figure (1), the nomograph. Q and P are the uncalibrated auxiliary scales.

# e. Scale Modulus or Unit of Representation

The scale modulus, M, is a "mapping factor" which fits the range of the variables on the sheet of paper containing the nomograph and establishes the separation distance between the vertical scales. The equation used to determine the scale modulus is, from reference (d):

Paper distance = M (upper range of variable)-(lower range of variable)
This distance has been utilized in making the nomograph.

#### USE OF THE NOMOGRAPH

Figure 1 is a plot of equations (1) through (4), in nomograph form. Equations (5) through (8), derived from equations (1) through (4), were used in making the nomograph. The pertinent symbols are defined both in the Glossary and in Figure 1. An example of the use of the nomograph is also shown and explained on Figure 1. Figure 2 shows the geometry of a towed system.

## Prior Information Needed Before Using Nomograph

The nomograph, Figure 1, will give the tension in, and the configuration for, the towcable. However, some other computations must be performed in order to obtain the non-dimensional cable functions that are needed, so that Figure 1 may be utilized. Reference (c) lists the information that is needed and illustrates the procedures that should be followed to obtain this information. It is suggested that both reference (c) and the present memorandum be used conjointly in solving towing problems.

#### Example

For a VDS towing operation, assume that the following information has been obtained by means of the precedures shown in reference (c):

- 1. Speed of tow = 20 knots;
- 2. Water weight of towed body =  $L_0 = 5562$  pounds;
- Horizontal drag force against towed body = D<sub>0</sub> = 1245 pounds;
- 4. Water weight of towline = W = 2.74 pounds/foot;
- 5. Cable drag of towline normal to stream = R = 20 pounds/foot;
- 6. Resultant tension at towstaff of towed body =  $T_0$  = 5700 pounds;
- 7. Towstaff angle,  $\tan^{-1} \frac{L_o}{D_o} = \phi = 77^\circ 25^\circ$ ;
- 8. Critical Angle of tow for towline =  $\phi \approx 20^{\circ}$ ; and
- 9. Length of cable payed out = S = 500 feet.

With the above information, it is found from reference (c) that the non-dimensional cable functions at the towstaff point, are:

$$\tau = 1.034$$

$$o = 0.238$$

$$\xi_0 = 0.027$$

$$\eta_0 = 0.236$$

The subscript zero refers to the towstaff point.

## Solution

(See Figure 1)

- Step 1. Lay a straightedge from the  $T_{\rm o}$  scale at 5700 pounds to the  $\tau_{\rm o}$  scale at 1.034.
- Step 2. Mark the point of intersection of the straightedge with the non-graduated "Q" scale.
- Step 3. Lay a straightedge from the point on the "Q" scale, step 2, to the "R" scale at 20 pounds/foot.
- Step 4. Mark the point of intersection of the straightedge with the non-graduated "P" scale.
- Step 5. Lay a straightedge from the S scale at 500 feet and the point of intersection on the "P" scale, step 4.
- Step 6. Mark the point of intersection on the  $\Delta \sigma$  scale. The value of  $\Delta \sigma = (\sigma \sigma_o) = 1.75$

Since  $\sigma = 0.238$ , the value of  $\sigma$  at the surface of the water is obtained as follows:

$$\Delta \sigma = (\sigma - \sigma_0) = 1.75 = \sigma - 0.238$$
,

or  $\sigma = 1.9888$ .

Referring again to reference (c), it is found that for  $\phi_c = 20^\circ$  and  $\sigma = 1.988$ , the surface towardle,  $\phi$ , is approximately 33°. Using  $\phi_c = 20^\circ$  and  $\phi = 33^\circ$ , from reference (c), the remaining functions are found to be:

$$\eta = 1.55$$

Therefore,

$$\Delta \xi = \xi - \xi_o = 1.19 - 0.027 = 1.16$$
  
 $\Delta \eta = \eta - \eta_o = 1.55 - 0.236 = 1.31$ 

- Step 7. Lay a straightedge from the  $\tau$  scale at 1.24, to the point of intersection on the "Q" scale, Step 2.
- Step 8. Mark the point of intersection with the T scale, 6900 pounds. This represents the tension in the cable at the surface of the water.
- Step 9. Lay a straightedge from the  $\Delta \xi$  scale at 1.16, to the point of intersection on the "P" scale, Step 4.
- Step 10. Mark the point of intersection with the X scale, 325 feet. This represents the trail distance of the towed body.
- Step 11. Lay a straightedge from the  $\Delta\eta$  scale at 1.31, to the point of intersection on the "P" scale, Step 4.
- Step 12. Mark the point of intersection with the Y scale, 365 feet.

  This represents the depth of the towed body.

### Summary of Calculations

At a towing speed of 20 knots, with a cable-payed-out length of 500 feet and a towstaff tension of 5700 pounds, the towcable tension at the surface is 6900 pounds, the trail distance of the towed body is 325 feet, and the depth of the towed body is 365 feet.

### CONCLUSION

A nomograph has been prepared that provides simple solutions of towed-body VDS problems. Provided that preliminary information on the characteristics of the towed body and towline are known, the nomograph facilitates computations of (1) towline tension at any particular point along the cable, and (2) the horizontal or vertical projected distances from the towed body to the point of interest. The towed body trail and depth can also be determined.

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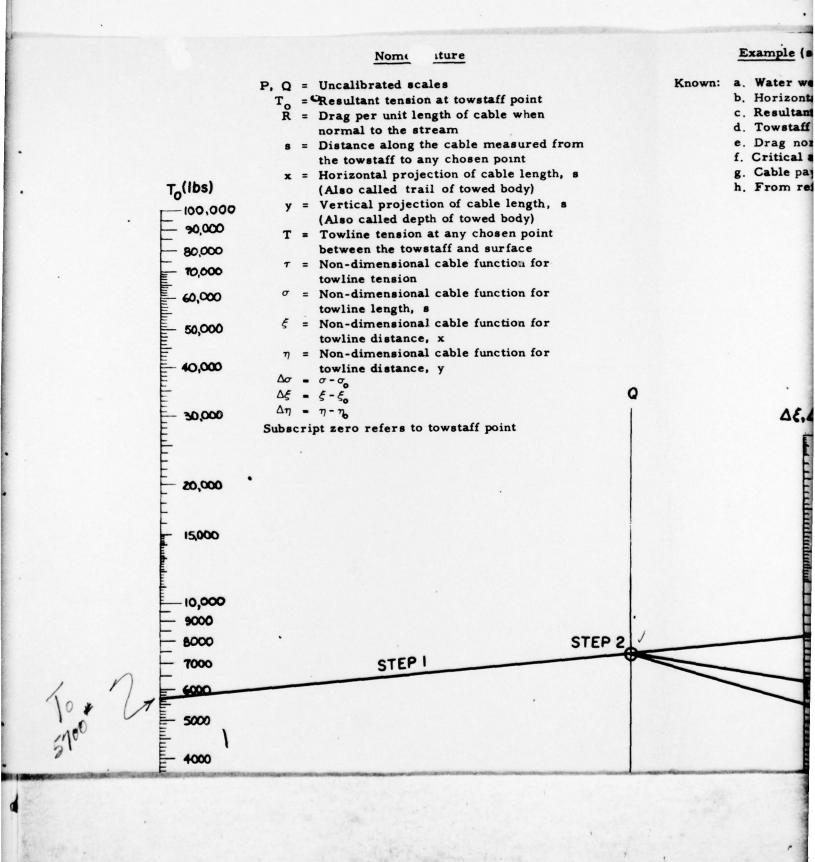
MATTHEW F. BORG Mechanical Engineer

# GLOSSARY OF TERMS

<u>Symbol</u>	<u>Definition</u>	Units
Do or FD	Horizontal Drag Force against towed body	lbs.
Lo	Water Weight of towed body	lbs.
М	Scale Modulus	in./unit dimension
P	Uncalibrated Scale	-
Q	Uncalibrated Scale	<u>-</u>
R	Cable Drag when cable is normal to direction of motion	lbs./ft.
S	Length of Cable from towed body attachment point to any point on the cable	ft.
T	Tension at any point on the cable	lbs.
To	Tension at towpoint (at towstaff)	lbs.
W	Non-dimensional ratio W/R	-
W	Waterweight of towline per unit leng	th lbs./ft.
X	Horizontal projected distance from towpoint to point of interest (trail when surface point is used, See Fig. 2)	ft.
¥	Vertical projected distance from towpoint to point of interest (depth when surface point is used, see Fig. 2)	ft.
7	Non-dimensional cable function asso ciated with Y distance	-
$\Delta \eta$	$\eta - \eta_{\circ}$	-
Ę	Non-dimensional cable function asso ciated with X distance	-
Δξ	ξ - ξ <sub>o</sub>	
σ	Non-dimensional cable function associated with S length	-
Δσ	$\sigma - \sigma_{\circ}$	-
τ	Non-dimensional cable function associated with T tension	
$\Delta  au$	$\tau - \tau_o$	
φ	Angle between tangent to the cable and the direction of motion at any point on the cable	degrees
$\phi_{e}$	Value of $\phi$ when cable (by itself) is towed freely ( $\phi_c = f(W/R)$ )	degrees

### List of References

- (a) Pode, L., "Tables for Computing the Equilibrium Configuration of a Flexible Cable in a Uniform Stream," DTMB Report No. 689, March 1951
- (b) Whicker, L. F., "The Oscillatory Motion of Cable-Towed Bodies," Ph.D. Dissertation, University of California, 16 July 1957
- (c) Borg, M.F., "Towed Body Solutions Using Graphs of the Non-Dimensional Cable Functions," USL Technical Memorandum No. 933-27-63 of 1 April 1963 (UNCLASSIFIED)
- (d) Johnson, L.H., "Nomography and Empirical Equations," John Wiley & Sons, Inc., New York, 1952.
- (e) DelSanto, R. F., Jr., "Variable Depth Sonar A Summary Report to 1962," USL Report No. 532, 4 January 1962 (CONFIDENTIAL)



e text for inplete explanation) Steps in using the nomograph ight of towed body = L<sub>O</sub> = 5562 pounds 1 drag force against towed body = D<sub>O</sub> = 1245 pounds Step 1. Lay a straightedge from the To scale at 5700 pounds to the 5 scale at 1.034 tension at towstaff = To = 5700 pounds Step 2. Mark the point of intersection of the straightedge with the nongraduated  $ext{angle} = \phi_0 = 77^{\circ}-25^{\circ}$ "Q" scale mal to the towline = R = 20 pounds/foot Lay a straightedge from the point on ngle of tow for towline =  $\phi_c \approx 20^{\circ}$ Step 3. ed out = s = 500 feet the "Q" scale, step 2, to the R erence (c), % = 1.034 scale at 20 pounds/foot Mark the point of intersection of the = 0.238 0 = 0.027 Tension Intrue of wares. straightedge with the nongraduated = 0.236 "P" scale Lay a straightedge from the s scale Step 5. at 500 feet and the point of intersection on the "P" scale, step 4 Step 6. Mark the point of intersection on the  $\Delta \sigma$  scale. The value of  $\Delta \sigma \simeq 1.75$ 7,00 T(Ibs) -0.01 1,000,000 800,000 600 poo 400,000 0.02 200,000 0.04 100,000 80,000 0,000 40,000 0.08 -0.1 20,000 STEP 4 10,000 8000 STEP 8

From reference (c), this  $\sigma$  corresponds to a  $\phi \approx 33^{\circ}$ The remaining functions are: an T = 1.244 rmediate \$ = 1.19 step  $\eta = 1.55$ Hence  $\Delta \xi = \xi - \xi_0 = 1.16$ R(1bs/ft)  $\Delta \eta = \eta - \eta_0 = 1.31$ Lay a straightedge from the  $\tau$  scale at 1.24 to the point of intersection on the "Q" scale, step 2 p 8. Mark the point of intersection with the T scale, 6900 pounds sp 9. Lay a straightedge from the Δξ scale at 1.16 to the point of intersection on the "P" scale, step 4 p 10. Mark the point of intersection with the x scale, 325 feet 02 sp 11. Lay a straightedge from the  $\Delta \eta$  scale at 1.31 to the point of intersection on the "P" scale op 12. Mark the point of intersection with the y scale, 365 feet - 0.3 - 04 0.5 0.6 0.7 0.8 0.9 EP12 Opth of T/B = 365 EPIO TONIL DIST.

The value of  $\sigma$  at the surface =  $\sigma = \Delta \sigma + \sigma_a = 1.988$ 

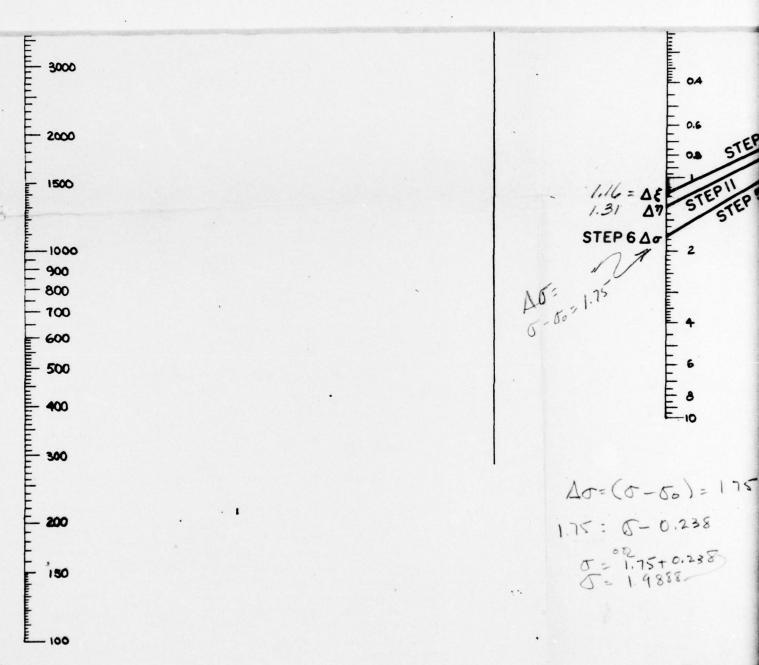
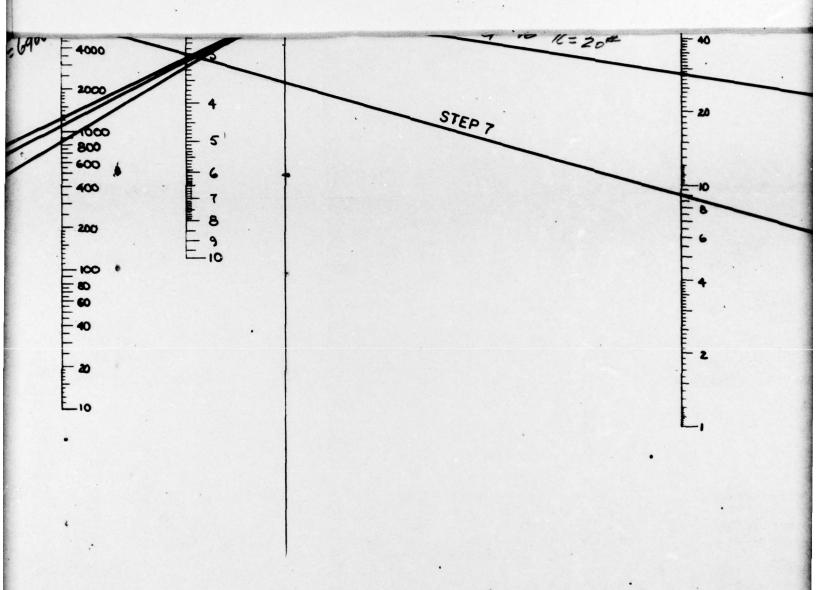
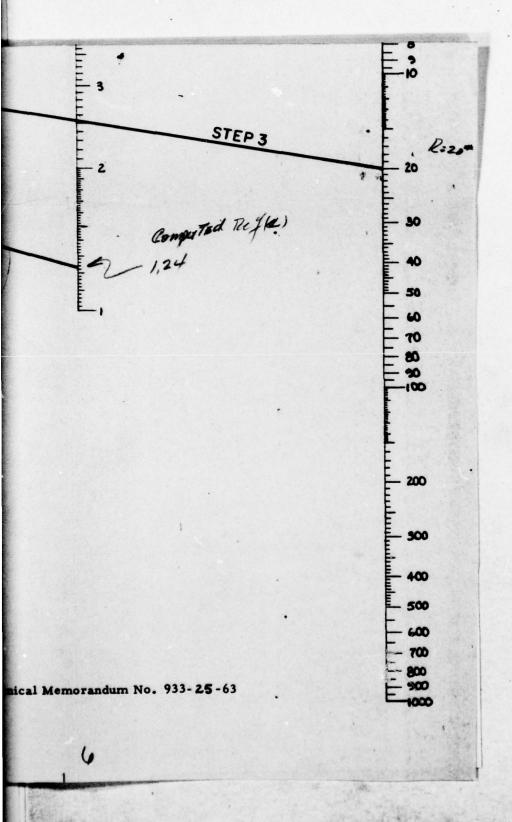


Fig. 1 - A 1



tograph for the Solution of Cable Characteristics for Steady State Towed Bodies

USL Ted



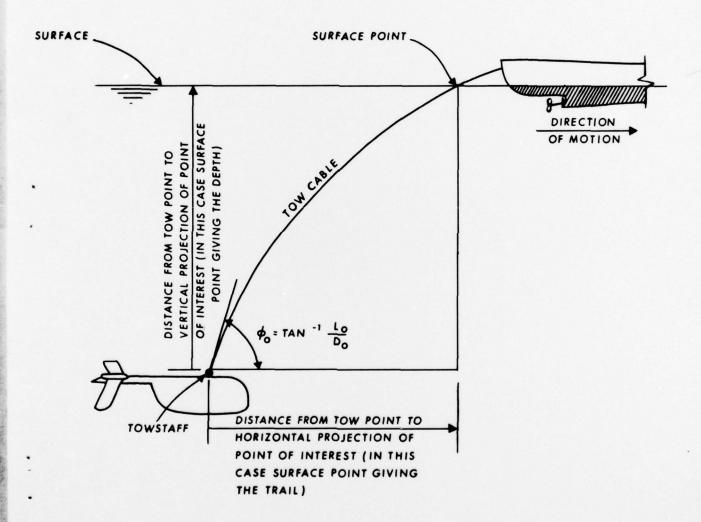


Fig. 2 - Geometric Orientation

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